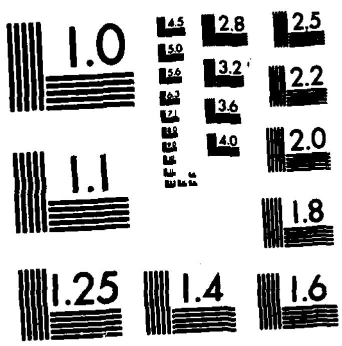
AD-A124 433 MODELS FOR FREQUENCY-HOPPED SPREAD-SPECTRUM COMMUNICATIONS OVER FADING CHANNELS(U) ILLINOIS UNIV AT URBANA COORDINATED SCIENCE LAB C M KELLER DEC 82 R-976 1/1

UNCLASSIFIED N00014-79-C-0424 F/G 17/2 NL

END

FILMED

DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

12

[illegible] NOVEMBER, 1982 UILU-ENG 82-2242

CSL COORDINATED SCIENCE LABORATORY

AD A124433

MODELS FOR FREQUENCY-HOPPED SPREAD-SPECTRUM COMMUNICATIONS OVER FADING CHANNELS

DTIC FILE COPY

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

83 02 015 012

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO. A124433	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) MODELS FOR FREQUENCY-HOPPED SPREAD-SPECTRUM COMMUNICATIONS OVER FADING CHANNELS		5. TYPE OF REPORT & PERIOD COVERED Technical Report
		6. PERFORMING ORG. REPORT NUMBER R-976 UILU-ENG 82-2242
7. AUTHOR(s) Catherine Marie Keller		8. CONTRACT OR GRANT NUMBER(s) N00014-79-C-0424
9. PERFORMING ORGANIZATION NAME AND ADDRESS Coordinated Science Laboratory University of Illinois at Urbana-Champaign Urbana, IL 61801		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Joint Services Electronics Program		12. REPORT DATE December 1982
		13. NUMBER OF PAGES 51
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Noncoherent Frequency-Hopped Communications
Fading Channel

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The system consisting of the series connection of a frequency-hopper, channel, and frequency-dehopper is modeled as a fading channel. The effect of a detuning factor between the hopping patterns of the hopper and dehopper is investigated. Both slow-frequency-hopped and fast-frequency-hopped systems are studied for cases when the fading channel is nonselective and selective. The ability of the frequency-hopped system to overcome frequency-selective fading is considered. Limits are found on the spacing between frequency slots in order that the fading in different slots is independent.

DD FORM 1 JAN 73 1473

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

MODELS FOR FREQUENCY-HOPPED SPREAD-SPECTRUM COMMUNICATIONS OVER FADING CHANNELS

by

Catherine Marie Keller

This work was supported by the Joint Services Electronics Program under Contract N00014-79-C-0424.

MODELS FOR FREQUENCY-HOPPED SPREAD-SPECTRUM COMMUNICATIONS OVER FADING CHANNELS

BY

CATHERINE MARIE KELLER

B.S., Carnegie-Mellon University, 1980

THESIS

Submitted in partial fulfillment of the requirements
for the degree of Master of Science in Electrical Engineering
in the Graduate College of the
University of Illinois at Urbana-Champaign, 1983

Thesis Adviser: Professor M. B. Pursley

Urbana, Illinois

MODELS FOR FREQUENCY-HOPPED SPREAD-SPECTRUM COMMUNICATIONS OVER FADING CHANNELS

by

Catherine Marie Keller
Department of Electrical Engineering
University of Illinois at Urbana-Champaign, 1983

ABSTRACT

The system consisting of the series connection of a frequency-hopper, channel, and frequency-dehopper is modeled as a fading channel. The effect of a detuning factor between the hopping patterns of the hopper and dehopper is investigated. Both slow-frequency-hopped and fast-frequency-hopped systems are studied for cases when the fading channel is nonselective and selective. The ability of the frequency-hopped system to overcome frequency-selective fading is considered. Limits are found on the spacing between frequency slots in order that the fading in different slots is independent.

ACKNOWLEDGEMENTS

The author is sincerely grateful to her advisor, Professor Michael B. Pursley, for his patient guidance and expert advice. Thanks are also extended to Mr. Fred D. Garber for his suggestions and encouragement.

Finally, thankful consideration is due to Mrs. Monica Ray and Mrs. Phyllis Young for their excellent typing.

TABLE OF CONTENTS

1. INTRODUCTION

Fading channels can cause severe degradation in the performance of digital communication systems. Many times this fading is due to multipath interference; that is, when a communication signal is transmitted, a superposition of several signals with different amplitudes and delays is received. Multipath interference arises in beyond-the-horizon communications when the transmitted signal is reflected from the tropospheric or ionospheric layers of the atmosphere [2]. It may also occur in communications when there are surrounding structures that become reflecting surfaces for the signals. If the relative delays of the received signals are long enough, i.e., longer than the duration of one data bit of information, the fading is frequency-selective.

Another type of fading comes about when either the transmitter or receiver are in motion or when the medium from which the signals reflect is moving. If the motion is rapid enough, i.e., faster than the duration of one data bit of information, the fading is time-selective.

Spread-spectrum communication systems offer aid in combatting the type of selective fading due to multipath. Spread-spectrum modulation achieves this by increasing the signal bandwidth so that it is greater than the coherence bandwidth of the fading channel. The coherence bandwidth of the channel is the separation between frequencies necessary for the fading at these frequencies to be independent [14].

In frequency-hopped (FH) spread-spectrum communication systems, the spreading of the signal spectrum is achieved by hopping the signal over a set of frequencies. The bandwidth of the FH system depends mainly on the spacing and the number of frequency slots to which the carrier is hopped.

For most systems of current interest, the bandwidth does not depend on the hopping rate. One reason the system tolerates rapid frequency-selective fading is that not all portions of the spectrum fade simultaneously [10]. Even though the fading may be selective across the frequency band used by the FH system, it may be nearly nonselective over each narrow-band channel. In this way, the FH system provides a form of frequency diversity. The FH system has total bandwidth much wider than the coherence bandwidth of the fading channel, but the system uses a smaller portion of the bandwidth at one time.

Probability of error analyses have been done for FH communications over fading channels. An analysis is presented in [9] for noncoherent binary-frequency-shift-keyed (FSK) slow-frequency-hopped (SFH) communications over fading channels. Both nonselective and selective fading is considered along with multiple-access interference. The system performance is shown to improve with coding. Coding for a binarv-phase-shift-keyed (BPSK) system that operates over fading channels is studied in [11]. In [7] and [8], analyses are done for noncoherent differential-PSK/SFH communications over time-selective and frequency-selective channels, respectively. Various pulse waveforms and various channel correlation functions are considered and comparisons are made among the resulting communication systems. The irreducible error probability due to the fading channel is found.

The purpose of this work is to look at a noncoherent FH communication system from a different point of view. We model the system consisting of the series connection of a frequency-hopper, channel, and frequency-dehopper as a fading channel seen by an FSK receiver. We analyze the role

that the FH system plays in either adding to the time-selective fading of the channel or in combatting frequency-selective fading. Both SFH and fast-frequency-hopped (FFH) systems are considered.

The frequency hopper and dehopper are each random time-varying systems. They are random because of a random phase introduced in each subsystem. They are time-varying because the frequency modulation in each changes with time. The fading channels we study may be described as linear systems whose impulse responses are time-varying random processes. Thus, the impulse response of the system consisting of the three systems in series may be described by some function that is a time-varying random process [3], [13], [19]. A model of the communication system is shown in Figure 1, where the impulse response $g(\cdot,\cdot)$ is a time-varying random process. In Section 2, we discuss the characterization of a system in terms of linear time-varying impulse responses and define a second-order characterization for responses that are random. We discuss the model for the FH system in Section 3 and find the time-varying impulse response for the frequency-hopper and frequency-dehopper.

The characteristics of the FH system operating over an ideal channel are investigated in Section 4. From this analysis we see that the FH system is a wide-sense-stationary (WSS) system that introduces time-selectivity. In Section 5 we analyze the system operating over nonselective channels. In Section 6 we study the fading channel that results when we have a frequency-hopper, a WSS uncorrelated-scattering (WSSUS) channel, and a frequency-dehopper in series. We investigate whether or not the composite system maintains the WSSUS property. Although we consider cases when the channel is either purely time-selective or purely frequency-selective, the purely

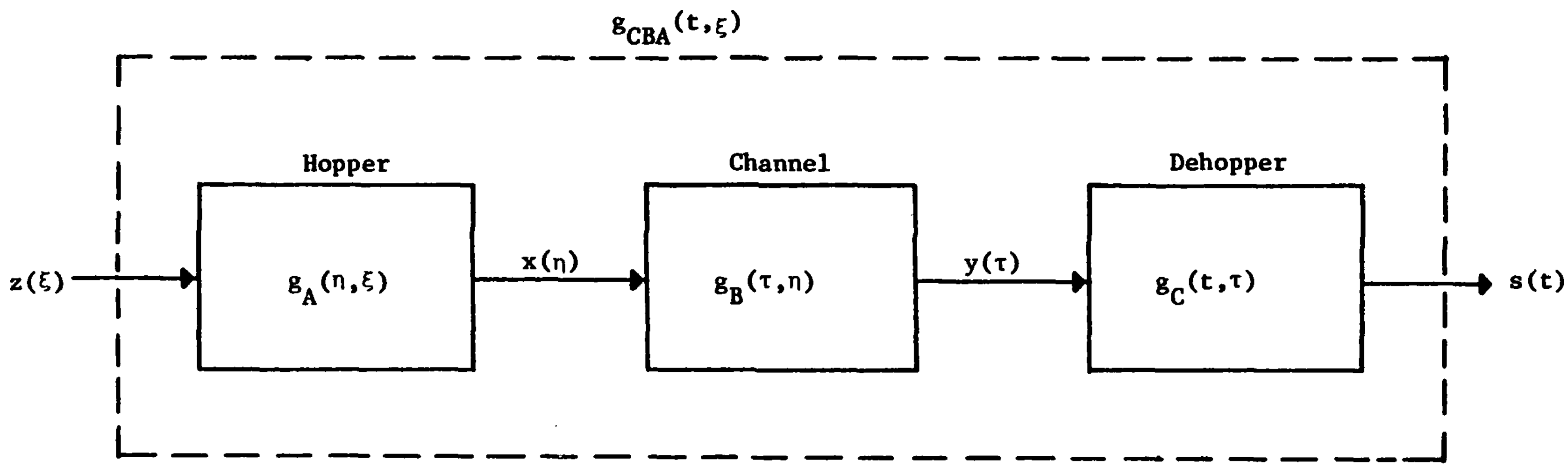


Figure 1. The system consisting of a frequency-hopper, channel, and frequency-dehopper viewed as a fading channel.

frequency-selective channel is of greatest interest [5], [8], [9], [11], [15], [18], [21]. We investigate the ability of the FH system to overcome frequency-selective fading.

Several papers consider frequency spacing between adjacent slots to be wide enough to assume independent fading from slot to slot [11], [15]. For example, the assumption is made in [11] that the spacing between adjacent frequency slots is large enough so that signals occupying different slots fade independently. In [15], the behavior of the overall channel is simulated by a group of independent narrowband channel models. The validity of this assumption is not established in previous investigations. In Section 7 we find the correlation in fading between frequency slots. This analysis finds limits on the spacing between frequency slots in order that the fading in different slots is independent.

2. CHARACTERIZATION OF LINEAR TIME-VARYING SYSTEMS

We describe our FH system using functions that are time-varying random processes. Bello [3] discusses a set of functions that are related to one another by Fourier transformations and a set of functions that are time-frequency duals to the first set. Any one of these functions can be used to describe a linear time-varying system as a fading channel. They are each derived from one of what Bello calls system kernel functions -- functions usually found in linear system theory (i.e., [24]) called time-varying impulse response and time-varying transfer function. We use the input delay-spread function $g(t,\xi)$, (see [3]) derived from the time-varying impulse response $H(t,\xi)$, to describe each of the three subsystems, frequency-hopper, channel, and frequency-dehopper.

The function $H(t,\xi)$ is defined as the response at time t due to an impulse applied at time ξ. All the signals are assumed to be narrowband signals. A signal is narrowband if the spectral components of the signal are restricted to a band that is small compared with the center frequency of the band. Given that the input to the system is the narrowband signal

$$z(\eta) = \mathrm{Re}\{w(\eta)\} = \mathrm{Re}\{v(\eta)\, e^{j2\pi f_c \eta}\} ,$$

where $v(\eta)$ is the complex envelope and f_c is the carrier frequency (spectral components of $v(\eta)$ are small compared to f_c), the system output is the real part of

$$d(t) = \int w(\eta) H(t,\eta)\, d\eta \quad . \tag{2.1}$$

Unless indicated otherwise, the lower limit on all integrals is $-\infty$ and the upper limit is $+\infty$. The causality condition is that $H(t,\eta) = 0$ for $\eta < t$.

To show that $H(t,\xi)$ is the correct impulse response, let $w(\eta) = \delta(\eta - \xi)$ in (2.1) and find

$$d(t) = \int \delta(\eta - \xi) H(t,\eta)\, d\eta = H(t,\xi) \quad .$$

Once we have the kernel function for each of the three subsystems, we find the kernel function for their series connection. For example, if we have a system consisting of two time-varying subsystems in series, say system A followed by system B, we find the overall time-varying response using the result [24]

$$H_{BA}(t,\xi) = \int H_B(t,\eta) H_A(\eta,\xi)\, d\eta \quad . \tag{2.2}$$

Noting which subsystem is first and which is second is important since in general the subsystems are not symmetric with respect to delay; i.e.,

$$\int H_B(t,\eta) H_A(\eta,\xi)\, d\eta \neq \int H_A(t,\eta) H_B(\eta,\xi)\, d\eta \quad .$$

The results for two subsystems in series are readily extended to three systems in series.

We use the system kernel function to obtain $g(t,\xi)$; this is the response at time t due to an impulse applied ξ time units in the past. We use $g(t,\xi)$ to characterize the system as a fading channel [3], [13]. Let $\eta = t - \xi$ in (2.1) to find that the system output is equivalently given as the real part of

$$d(t) = \int w(t - \xi) H(t,\, t-\xi)\, d\xi$$

and define

$$g(t,\xi) = H(t,\, t-\xi) \quad . \tag{2.3}$$

To check that this is the correct response, let $w(\eta) = \delta(t - \eta - \xi)$ in (2.1). The signal out is $d(t) = g(t,\xi)$.

We can give the integral

$$d(t) = \int w(t - \xi)\, g(t,\xi)\, d\xi \tag{2.4}$$

a physical interpretation so that the system appears to be a fading channel. We say that the channel consists of a continuum of scattering layers with elemental thickness that produces a complex modulation $g(t,\xi)\, d\xi$ and causes delays in the range $(\xi, \xi+d\xi)$. The input signal is first delayed and then multiplied by the differential weighting function. The output is a sum of all the contributions. The delay parameter ξ measures the age of the input. The causality condition is that $g(t,\xi) = 0$ for $\xi < 0$ and reflects that the channel cannot weight portions of the input that have not yet occurred. The overall input delay-spread function for two systems in series, A followed by B, is

$$g_{BA}(t,\xi) = H_{BA}(t,\ t-\xi) = \int g_B(t,\ t-\eta) g_A(\eta,\ \eta-(t-\xi))\, d\eta \ .$$

We first find $H_{BA}(t,\xi)$ using (2.2) and then use the relationship between g and H in (2.3) to find $g_{BA}(t,\xi)$.

The function $g(t,\xi)$ is a random process if the system it describes has some random parameter. As we see in the system model description in Section 3, this is the case for our frequency-hopping communication system. Thus, to describe the system further, we need some characterization of the system's random behavior. We use the autocorrelation function [3]

$$R_g(t,s;\ \xi,\zeta) = E\{g^*(t,\xi) g(s,\zeta)\}\ , \tag{2.5}$$

which is a second order characterization. In dealing with fading channels, there are two things we look for when examining the system's autocorrelation function. If the function depends on time difference $\kappa = s - t$ only, then we write

$$R_g(t, t+\kappa; \xi,\zeta) = R_g(\kappa; \xi,\zeta) \quad , \tag{2.6}$$

and the system is said to be wide-sense-stationary (WSS). If the channel is characterized by uncorrelated-scattering (US)--that is, delays and modulation produced by one layer of scatterers are uncorrelated with delays and modulation produced by any other layer of scatterers--then there is a corresponding mathematical form for R_g; i.e., there is a function P_g that satisfies

$$R_g(t, t+\kappa; \xi,\zeta) = P_g(t, t+\kappa; \xi)\delta(\xi-\zeta) \quad . \tag{2.7}$$

This says that $g(t,\xi)$ and $g(t+\kappa;\zeta)$ are uncorrelated whenever $\xi \neq \zeta$. The frequency-hopping system itself cannot exhibit the US property in the physical sense since there are no real scattering layers that reflect input signals. However, the mathematical form for an US channel may be useful. If the channel is WSS and US then

$$R_g(t, t+\kappa; \xi,\zeta) = P_g(\kappa;\xi)\delta(\xi-\zeta) \quad . \tag{2.8}$$

It is possible to classify WSSUS fading channels into four groups: time-selective, frequency-selective, non-selective, and doubly-selective [3], [14]. If $P_g(\kappa,\xi)$ can be written as

$$P_g(\kappa;\xi) = P_g(\kappa,0)\delta(\xi) \quad , \tag{2.9}$$

then the channel is time-selective (frequency-dispersive). Time-selective fading means that a signal sent over the channel is attenuated by a time-varying gain and the spectrum of the signal is spread according to how fast the attenuation changes. This may give rise to different signals having overlapping spectra.

Define the Fourier transform,

$$\mathcal{P}(f) \triangleq \int P(\kappa)\ e^{-j2\pi f\kappa}\ d\kappa \tag{2.10}$$

where we let $P(\kappa) \triangleq P_g(\kappa;0)$. This is the power spectrum for the system function g that we use to calculate the channel mean Doppler shift m_p, the shift of the center or maximum of $\mathcal{P}(f)$ from the $f = 0$ axis, and the channel mean Doppler spread D_p, the spectral "width" of the power spectrum. A calculation for mean Doppler shift is [4], [18]

$$m_p = \frac{\int f\mathcal{P}(f)\ df}{\int \mathcal{P}(f)\ df}$$

and for mean Doppler spread we may use a measurement such as 3 dB bandwidth, defined as the smallest interval between frequencies at which $\mathcal{P}(f)$ drops to half its maximum value. If $\mathcal{P}(f)$ has well defined nulls, we may use the null-to-null bandwidth defined as the width of the main lobe of $\mathcal{P}(f)$ [2],[5].

The dual to the time-selective channel is the frequency-selective (time-dispersive) channel. Then $P_g(\kappa;\xi)$ is written as

$$P_g(\kappa;\xi) = P_g(0;\xi) \triangleq Q(\xi) \quad . \tag{2.11}$$

A frequency-selective fading channel attenuates certain frequency components of a transmitted signal so that the signal is spread in time. The

problem of intersymbol interference may be introduced by the channel for successive signals sent. Using $Q(\xi)$, called the delay density spectrum, we calculate the channel mean path delay m_Q and the channel mean multipath spread M_Q using definitions dual to the time-selective calculations.

Nonselective fading can be described completely as a multiplicative complex process. It does not introduce pulse lengthening which can cause intersymbol interference nor does it introduce Doppler spreading which can cause overlapping spectra for signals in adjacent frequency slots. We consider slow fading; i.e., fading that can be regarded as constant over the duration of several data bits [21]. In this case we have

$$P_g(\kappa,\xi) = P_g(0,0)\delta(\xi) \quad . \tag{2.12}$$

If $P_g(\kappa,\xi)$ is a function of both κ and ξ that cannot be written in any of the above forms, then the channel is doubly-selective [14]. A signal sent through this type of channel is spread in both time and frequency so that in data communication there is danger of overlapping spectra as well as intersymbol interference. The mean Doppler shift and mean Doppler spread are calculated using the power spectral density

$$\mathcal{P}(f) = \int \hat{P}_g(\kappa,0)\; e^{-j2\pi f\kappa}\, d\kappa \tag{2.13}$$

where

$$\hat{P}_g(\kappa,\Omega) = \int P_g(\kappa,\xi)\; e^{-j2\pi\Omega\xi}\, d\xi \; .$$

As before, $Q(\xi) = P_g(0,\xi)$, and we use this to measure the mean path delay and multipath spread.

3. SYSTEM MODEL

We consider an FH system with one transmitter and one receiver. The transmitter is the same as in [9]. The input to the hopper is an FSK signal with carrier frequency f_c,

$$z(\eta) = \mathrm{Re}\{u(\eta)\ e^{j2\pi b(\eta)\Delta\eta}\ e^{j2\pi f_c \eta}\} = \mathrm{Re}\{w(\eta)\}$$

where $b(\eta)$ is the data signal which is a sequence of rectangular pulses each with amplitude +1 or -1 and of duration T. The constant Δ is one half of the frequency spacing between the two FSK tones. The complex envelope $u(\eta)$ is due to a phase introduced at the FSK modulator that depends on the value of the data pulse at time η. Note that the complex envelope of the input signal $v(\eta) = u(\eta)\ e^{j2\pi b(\eta)\Delta\eta}$ is to be slowly varying when compared to $1/f_c$ for the signal to be considered narrowband. The bandwidth of a sequence of pulses is approximately $2(\Delta + c/T)$ where c is a constant depending on what bandwidth measurement we use; e.g., $c = 1$ for the null-to-null bandwidth, so that the FSK signals are narrowband if $f_c \gg 2(\Delta + 1/T)$.

The FSK signal is frequency hopped according to a hopping pattern which is a sequence of frequencies $(f_j) = \ldots, f_{-1}, f_0, f_1, \ldots$. We define the frequency hopping function $f_h(\eta)$ such that $f_h(\eta) = f_j$ for $jT_h \le \eta < (j+1)T_h$ where T_h is called the dwell time. For SFH T_h is an integer multiple of the data bit duration, and for FFH T is an integer multiple of T_h. We assume that the hopping sequence is synchronized with the data bit sequence so that for SFH the start of a hopping interval is also the start of a data bit interval and visa-versa for FFH.

There is a phase shift $\alpha(\eta)$ introduced when the hopper switches from one frequency to another [9, p. 997]. We consider it to be a random process

such that during each hopping interval it is a random variable uniformly distributed on $[0,2\pi)$ independent of $\alpha(\eta)$ in any other hopping interval. The bandpass filter removes the unwanted frequency components present at the output of the multiplier. We assume that only the difference frequency is passed.

The output signal of the hopper is

$$x(\eta) = \mathrm{Re}\{\frac{1}{2} w(\eta)\ e^{-j2\pi f_h(\eta)\eta - j\alpha(\eta)}\} . \tag{3.1}$$

To find the response $H_A(\eta,\xi)$ of the frequency-hopper at time η due to an impulse applied at time ξ, let $w(\eta) = \delta(\eta-\xi)$ in (3.1). The resulting output complex envelope is

$$H_A(\eta,\xi) = \frac{1}{2}\ \delta(\eta - \xi)\ e^{-j2\pi f_h(\eta) - j\alpha(\eta)} . \tag{3.2}$$

If the complex input signal is $w(\xi) = v(\xi)\ e^{j2\pi f_c \xi}$ then the output signal from the frequency-hopper is

$$x(\eta) = \mathrm{Re}\{\int H_A(\eta,\xi) w(\xi)\ d\xi\} .$$

The frequency-dehopper model is also in [9]. The received signal is frequency hopped according to the frequency hopping function $f_d(t)$. That is, for all t, $f_d(t) = f_h(t) + \varepsilon$, where ε represents a frequency offset that we call the detuning factor. The detuning factor ε is zero except in Sections 4.2 and 4.3. The phase shift $\beta(t)$ is introduced at the dehopper and is analogous to $\alpha(t)$. We consider modulo 2π addition of the phases α and β. Since there is a random phase introduced by the FH system at the start of each hop, the system is noncoherent. The bandpass filter is centered about f_c and has a bandwidth W that is greater than $2(\Delta + 1/T)$,

but less than the smallest spacing between hopping frequencies. (see [9, p. 998].) We call the time-invariant impulse response of the bandpass filter $h_W(t)$. For example, the response of an ideal bandpass filter is

$$h_W(t) = 2W \operatorname{sinc}(Wt) \cos(2\pi f_c t) \tag{3.3}$$

where $\operatorname{sinc}(x) = \sin(\pi x)/(\pi x)$. We assume that the bandpass filter is ideal or that it has frequency response that is nearly flat over the passband W. The impulse response of the dehopper is

$$\begin{aligned} H_c(t,\tau) &= \frac{1}{2}\,\delta(\nu-\tau)\, e^{j2\pi f_d(\nu)\nu+j\beta(\nu)}\, h_W(t-\nu)\, d\nu \\ &= \frac{1}{2}\, e^{j2\pi f_d(\tau)\tau+j\beta(\tau)}\, h_W(t-\tau) \quad . \end{aligned} \tag{3.4}$$

4. NONFADING CHANNEL

Suppose that the FH system operates over an ideal (infinite bandwidth, nonfading) channel. The input signal enters the hopper and after transmission it is received at the dehopper without additive or multiplicative interference. The overall system impulse response found by using (3.2) and (3.4) is

$$\begin{aligned} H_{CA}(t;\xi) &= \int H_C(t,\eta)H_A(\eta,\xi)\, d\eta \\ &= \frac{1}{4}\, e^{-j2\pi f_h(\xi)\xi - j\alpha(\xi)}\, e^{j2\pi f_d(\xi)\xi + j\beta(\xi)}\, h_W(t-\xi)\,. \end{aligned}$$

The response at time t due to an impulse applied ξ time units in the past is

$$\begin{aligned} g_{CA}(t,\xi) &= H_{CA}(t,\, t-\xi) \\ &= \frac{1}{4}\, e^{-j2\pi f_h(t-\xi)(t-\xi) - j\alpha(t-\xi)}\, e^{j2\pi f_d(t-\xi)(t-\xi) + j\beta(t-\xi)}\, h_W(\xi). \end{aligned} \tag{4.1}$$

4.1 Zero Detuning Factor

In this section we assume that the hopper and dehopper patterns are synchronized in time and frequency so that $f_d(t) = f_h(t)$ for all t. Then (4.1) becomes

$$g_{CA}(t,\xi) = \frac{1}{4}\, \delta(\xi)\, e^{-j\alpha(t)+j\beta(t)}\;, \tag{4.2}$$

where we have used the fact that

$$f(\xi)\delta(\xi) = f(0)\delta(\xi)\;, \tag{4.3}$$

where $f(\xi)$ is a function that is continuous at $\xi = 0$ and where we note that the expression containing the delta function is inside an integral

when we calculate the system output. Namely,

$$s(t) = \mathrm{Re}\left\{ \int g_{CA}(t,\xi)\ w(t-\xi)\ d\xi \right\} .$$

The form for $g_{CA}(t,\xi)$ is one of the simplest for a time-varying system since it can be written as

$$g_{CA}(t,\xi) = g_{CA}(t)\delta(\xi) .$$

This type of impulse response is characteristic of a complex amplitude modulator. The FH system is a phase modulator since the output is given by

$$s(t) = \mathrm{Re}\{g_{CA}(t)w(t)\} \tag{4.4}$$

where

$$g_{CA}(t) = \frac{1}{4}\ e^{-j\alpha(t)+j\beta(t)}$$

is the modulation. The autocorrelation function of the system is

$$R_{g_{CA}}(t,s;\ \xi,\zeta) = \frac{1}{16}\ \delta(\xi)\delta(\zeta)\ E\{e^{j\alpha(t)-j\beta(t)-j\alpha(s)+j\beta(s)}\} .$$

The expectation is found by making the following assumptions:

(i) The quantity $\theta(t) = \alpha(t) - \beta(t)$ is uniformly distributed on $[0,2\pi)$ in each hopping interval and is independent of $\theta(t)$ in any other hopping interval.

(ii) There is a random starting time for the hopping patterns. Call this t_0 and assume that it is uniformly distributed on $[0,T_h)$ and is independent of $\theta(t)$.

Other assumptions are possible for $\alpha(t)$ and $\beta(t)$, but assumption (i) as stated is a property of a noncoherent FH system and is of most interest. If $\alpha(t)$ and $\beta(t)$ are independent, each uniformly distributed on $[0,2\pi)$ in each hopping interval, and if the difference $\alpha(t) - \beta(t)$ is taken modulo 2π, then note that (i) follows.

After solving for the expectation, we have

$$R_{g_{CA}}(t,s;\ \xi,\zeta) = \begin{cases} \frac{1}{16}\delta(\xi)\delta(\zeta)\left[1 - \frac{|s-t|}{T_h}\right]; & |s-t| \le T_h \\ 0 & ;\ \text{otherwise} \end{cases} .$$

The product $\delta(\xi)\delta(\zeta)$ is interpreted as a unit mass at the point (0,0). It is non-zero only when both $\xi = 0$ and $\zeta = 0$ [16]. An equivalent expression is

$$\delta(\xi)\delta(\zeta) = \delta(\xi)\delta(\xi-\zeta) = \delta(\zeta)\delta(\xi-\zeta) \tag{4.5}$$

where we remember that when using $R_{g_{CA}}(t,s;\ \xi,\zeta)$ in calculations, say for calculating the autocorrelation function of the system output, the product of delta functions always appears under a double integral taken with respect to ξ and ζ. Thus, the system autocorrelation function may be written as

$$R_{g_{CA}}(t,s;\ \xi,\zeta) = \begin{cases} \frac{1}{16}\delta(\xi)\delta(\xi-\zeta)\left[1 - \frac{|s-t|}{T_h}\right] & ;\ |s-t| \le T_h \\ 0 & ;\ \text{otherwise} \end{cases} , \tag{4.6}$$

which depends on t and s only through the difference $\kappa = s - t$; it is a correlation function for a WSSUS channel. In addition, the system is purely time-selective since

$$P_{g_{CA}}(\kappa;\xi) = P_{CA}(\kappa)\delta(\xi)$$

where

$$P_{CA}(\kappa) = \begin{cases} \frac{1}{16}\left[1 - \frac{|\kappa|}{T_h}\right] & ; \ |\kappa| \le T_h \\ \\ 0 & ; \ \text{otherwise} \end{cases} \tag{4.7}$$

So, from the point of view of the FSK transmitter and receiver, the FH system operating over an ideal channel looks like a purely time-selective WSSUS fading channel. This is a reasonable result since purely time-selective implies that impulses applied to the system far enough apart in time will have uncorrelated fading. This time separation is called the correlation duration for the fading channel and for our system it is the dwell time T_h. The uncorrelated fading of signals sent at times separated by more than T_h is a result of assumptions (i) and (ii) on $\theta(t)$ and t_0. Thus, it is the noncoherent nature of the system that causes the time-selectivity.

The channel model we have discussed does not indicate the effect that the relationship between the dwell time and the bit duration has on the time-selectivity of the system. Also, it does not separate the case when the hopping sequence and the FSK data sequence are synchronized from the case when they are not. If the sequences are not synchronized, time-selectivity is introduced by the FH system. The spectrum of the sequence at the output of the channel is spread in frequency since, as we can see mathematically, the output spectrum is the result of a convolution of the input spectrum with the channel power spectral density. The operation is convolution, because as in (4.4), we see that the channel is a time-varying

complex multiplier in the time domain. The spectrum bandwidth of the output of the dehopper demodulator is less than

$$\frac{c}{T_h} + \frac{c}{T} + 2\Delta ,$$

where c is a constant that depends on what definition of bandwidth we use. Thus the Doppler spread is less than c/T_h. (A more exact calculation may be made by convolving $T \operatorname{sinc}^2(Tf)$ with $T_h \operatorname{sinc}^2(T_h f)$ and finding the resulting bandwidth.)

In the system model (Section 3), we assume that the hopping sequence and the modulated data sequence are synchronized. Assume that T_h/T is an integer and that the beginning of a hopping interval is also the beginning of an FSK data bit. Within any given hopping interval, the FH system does not look time-selective. Each data bit signal is multiplied by a random complex multiplicative factor that remains constant for the dwell time. However, over a longer period of time than T_h, some time-selectivity is exhibited. This is because of the random phases introduced by the FH system, which give rise to a Doppler shift on the order of 1 Hz. That is, each bit is multiplied by a complex factor $\frac{1}{4} e^{j\theta_n}$ causing a Doppler shift of $\theta_n/2\pi$ Hz. All the FSK data bits within the nth hopping interval have this same complex modulation, but the spectra of bits in other hopping intervals have different shifts. The output spectrum of the sequence of FSK data bits is spread on the order of 1 Hz since it is the superposition of many individual spectra, each shifted to slightly different frequencies. The mean Doppler shift is 0 Hz.

In fast-frequency-hopping, when $T > T_h$, the fading due to the

frequency-hopping does not remain constant for the duration of an FSK data bit. With respect to each input signal of duration T, the FH system looks like a time-selective channel. Its spectrum is spread and its waveform is distorted due to several random phase shifts being introduced during its transmission. For example, if the hopping sequence and bit sequence are synchronized in the sense that the start of a data bit is also the start of a hopping interval and T/T_h is an integer, then the spectrum of the output envelope of one bit will be about T/T_h times as wide as the spectrum of its input envelope. (If we assume that the FH system is coherent--that is, no phase shifts are introduced by the system--then the spreading of the signal spectrum does not occur. In [10] it is found that the probability of error of the noncoherent system increases when T/T_h increases, but in the coherent system, the probability of error does not depend on T/T_h.) If the Doppler spread is interpreted to be the difference between the bandwidth of the output spectrum of one FSK pulse and the bandwidth of the input spectrum of one FSK pulse, then

$$D_p \approx \frac{c}{T_h} - \frac{c}{T} = \frac{c}{T}\left(\frac{T}{T_h} - 1\right) .$$

There is an additional spread on the order of 1 Hz due to the values of the several random phases introduced within each data bit. Larger hopping rates may introduce larger additional spread than slower hopping rates, but the spread is still on the order of 1 Hz. Also, each data signal has a Doppler shift that is an average of the shifts caused by these random phases. When we look at the spectrum of a sequence of many data bits, we see that the mean Doppler shift is zero, while the additional spread of about 1 Hz remains.

Thus, in general, when the sequences are synchronized in one of the manners discussed above, the Doppler spread of an input FSK data sequence caused by the FH system may be written as

$$D_p \simeq \max\left(0, \frac{c}{T}\left(\frac{T}{T_h} - 1\right)\right) + O(1\ \text{Hz}) \tag{4.8}$$

where by O(1 Hz) we mean, on the order of 1 Hz.

The spacing between frequency slots and the frequency spacing Δ between two FSK tones must be larger for FFH systems than for SFH systems to accommodate the spread of signal spectra.

4.2 Constant Detuning Factor

Suppose that $f_h(t)$ and $f_d(t)$ are synchronized in time, but that there is some frequency offset between them. That is, $f_d(t) = f_h(t) + \varepsilon$ where ε is the detuning factor. In this section we consider when the detuning factor is constant. The input delay-spread function becomes

$$g_{CA}(t,\xi) = \frac{1}{4}\,\delta(\xi)\,e^{-j2\pi\varepsilon t - j\theta(t)}.$$

Using the noncoherent hopping assumptions from Section 4.1, we find that the autocorrelation function is

$$R_{g_{CA}}(t,s;\,\xi,\zeta) = \begin{cases} \frac{1}{16}\,\delta(\xi)\delta(\xi-\zeta)\,e^{-j2\pi\varepsilon(s-t)}\left[1 - \frac{|s-t|}{T_h}\right] & ;\ |s-t| \le T_h \\ 0 & ;\ \text{otherwise.} \end{cases} \tag{4.9}$$

Again, the system looks like a purely time-selective WSSUS channel in the same way as the system with zero detuning factor, but now,

$$P_{CA}(\kappa) = \begin{cases} \frac{1}{16} e^{-j2\pi\varepsilon\kappa} \left[1 - \frac{|\kappa|}{T_h}\right] & ; \ |\kappa| \le T_h \\ 0 & ; \ \text{otherwise} \end{cases}$$

so that there is a mean Doppler shift

$$m_p = -\varepsilon = \frac{\int f\mathcal{P}(f)\, df}{\int \mathcal{P}(f)\, df}$$

where

$$\mathcal{P}(f) = \frac{T_h}{16} \operatorname{sinc}^2 T_h(f + \varepsilon) \quad .$$

Thus, a constant detuning factor between the hopper and dehopper frequencies causes a constant Doppler shift equal in magnitude to this factor.

4.3 Frequency Dependent Detuning Factor

More realistically, the detuning factor between $f_h(t)$ and $f_d(t)$ is a function of frequency. For this analysis, we let $f_d(t) + f_c = (f_h(t) + f_c)[1 + \lambda]$ so that the magnitude of the frequency offset is a fraction λ of the instantaneous frequency of the signal. That is, the offset is a fraction of the hopping frequency plus the center frequency. (However, we can let $f_d'(t) = f_h'(t)[1 + \lambda]$ where f_c is incorporated in the hopping frequency functions. That is, we can let $f_h'(t) = f_h(t) + f_c$.) The fraction λ is a random process, independent of $\theta(t)$ and the random starting time t_0, but we assume that it varies slowly so that it can be thought of as constant over several hopping intervals. The input delay-spread function for the FH system is

$$g_{CA}(t,\xi) = \frac{1}{4}\,\delta(\xi)\,\exp[-j2\pi\lambda(f_h(t)+f_c)t - j\theta(t)]$$

and the autocorrelation function is

$$R_{g_{CA}}(t,s;\xi,\zeta) = \frac{1}{16}\,\delta(\xi)\delta(\xi-\zeta)\sum_{n=-\infty}^{\infty} e^{-j2\pi\lambda(s-t)f_n}\,E\{p_{T_h}(t-t_0-nT_h)p_{T_h}(s-t_0-nT_h)\},$$

where $p_{T_h}(t)$ is 1 for $t \in [0,T_h]$ and is 0 otherwise. The expectation is

$$\frac{1}{T_h}\int_{nT_h}^{(n+1)T_h} p_{T_h}(t-x)p_{T_h}(s-x)\,dx$$

$$= \frac{1}{T_h}\int_{t-(n+1)T_h}^{t-nT_h} p_{T_h-|s-t|}(u + \min(0,s-t)\,du \quad ; \quad |s-t| \le T_h\,.$$

The integral for the expectation is non-zero for at most two consecutive values of n. Thus, $R_{g_{CA}}$ consists of at most two terms and λ is nearly constant over the two hopping intervals involved because of our assumption. Let k-1 and k denote the values of n when the expectation is not zero. Then, we find that the autocorrelation is

$$R_{g_{CA}}(t,s;\xi,\zeta) = \begin{cases} \frac{1}{16}\delta(\xi)\delta(\xi-\zeta)\frac{1}{T_h}\Big[(s-kT_h)e^{-j2\pi\lambda f_k(s-t)} + (t-2s+(k+1)T_h) \\ \qquad \cdot e^{-j2\pi\lambda f_{k-1}(s-t)}\Big]; \; 0<s-t\le T_h \text{ and } \frac{2s-t-T_h}{T_h} \le k \le \frac{s}{T_h} \\ \frac{1}{16}\delta(\xi)\delta(\xi-\zeta)\frac{1}{T_h}\Big[(t-kT_h)e^{-j2\pi f_k(s-t)} + (s-2t+(k-1)T_h) \\ \qquad \cdot e^{-j2\pi\lambda f_{k-1}(s-t)}\Big]; \; -T_h\le s-t\le 0 \text{ and } \frac{2t-s-T_h}{T_h} \le k \le \frac{t}{T_h} \\ 0 \qquad ; \text{ otherwise}\,. \end{cases} \tag{4.10}$$

We note that if $\varepsilon = \lambda f_n$ for all n, then (4.10) reduces to the autocorrelation function for the constant detuning factor in (4.9). We notice from (4.10) that when the detuning factor is a function of frequency, the system is no longer WSS. At the start of each dwell time, a new frequency offset is introduced. For each value of s and t such that $|s-t| < T_h$, there is an integer k that falls in one of the intervals noted in (4.10) such that the autocorrelation function is non-zero. At most two adjacent frequencies are involved in (6.10) for any t and s (when $|s-t| < T_h$). That is, we can vary s and t (so that $|s-t|$ remains less than T_h) and consequently find the autocorrelation function involving any two adjacent hopping intervals.

We now impose a probability distribution on f_n that makes the system WSS. Assume that the hopping frequencies are independent, each uniformly distributed on the set of q available frequencies $\{\chi_i = f_c + \nu_i \mid \nu_i \in \{\nu_1, \nu_2, \ldots, \nu_q\}\}$. Note that this assumption makes it possible for $f_n = f_{n+1}$; i.e., frequencies in adjacent hopping intervals may be equal. We also assume that the frequencies are independent of $\theta(t)$ and t_0. Then we have that

$$E\{e^{-j2\pi\lambda f_k(s-t)}\} = E\{e^{-j2\pi\lambda f_{k-1}(s-t)}\} = \frac{1}{q}\sum_{i=1}^{q} e^{-j2\pi\lambda\chi_i(s-t)}$$

and the autocorrelation function is

$$R_{g_{CA}}(t,s;\xi,\zeta) = \begin{cases} \frac{1}{16}\,\delta(\xi)\delta(\xi-\zeta)\left[1 - \frac{|s-t|}{T_h}\right]\frac{1}{q}\sum_{i=1}^{q} e^{-j2\pi\lambda\chi_i(s-t)}; & |s-t| \le T_h \\ 0 & ;\ \text{otherwise.} \end{cases}$$

The Fourier transform of $P_{CA}(s-t)$ is the power spectral density

$$\mathcal{P}(f) = \frac{1}{16q} \sum_{i=1}^{q} T_h \; \mathrm{sinc}^2(T_h(f + \lambda\chi_i)),$$

which we use to calculate the mean Doppler shift and spread. The mean Doppler shift for the system is

$$m_p = \int f \sum_{i=1}^{q} T_h \mathrm{sinc}^2(T_h(f+\lambda\chi_i)) \; df \Big/ \int \sum_{i=1}^{q} T_h \mathrm{sinc}^2(T_h(f+\lambda\chi_i)) \; df$$

$$= -\frac{\lambda}{q} \sum_{i=1}^{q} \chi_i \; ,$$

which is the average of the available frequencies multiplied by $-\lambda$. The mean Doppler Spread is the "width" of $\mathcal{P}(f)$. It is the spread as discussed in Section 4.1 plus additional spread on the order of λ times the difference between the maximum available frequency and the minimum available frequency. The spread increases with λ.

5. NONSELECTIVE FADING CHANNELS

5.1 Rayleigh Fading Channel

A signal entering a nonselective Rayleigh fading channel is received as undistorted except for a Gaussian distributed multiplicative factor [21]. Call this factor $\rho e^{j\gamma}$ where ρ has a Rayleigh density

$$f_\rho(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} \quad ; \quad r \ge 0 \tag{5.1}$$

and γ is uniformly distributed on $[0,2\pi)$. For the FH system operating over this channel, the input delay-spread function is

$$g_{CBA}(t,\xi) = \frac{1}{4} \rho e^{-j2\pi(f_h(t-\xi)-f_d(t-\xi))(t-\xi)+j\beta(t-\xi)-j\alpha(t-\xi)+j\gamma} h_W(\xi).$$

Assume that $f_h(t) = f_d(t)$ for all t and that ρ and γ are independent of $\alpha(t)$ and $\beta(t)$ where the same assumptions, (i) and (ii), are made on $\alpha(t)$ and $\beta(t)$ as before. The autocorrelation function for the system is

$$R_{g_{CBA}}(t,s;\xi,\zeta) = \begin{cases} \frac{1}{16} \delta(\xi)\delta(\xi,\zeta)\{\rho^2\} \left[1 - \frac{|s-t|}{T_h} \right] & ; |s-t| \le T_h \\ 0 & ; \text{otherwise} \end{cases} \tag{5.2}$$

where $E\{\rho^2\} = 2\sigma^2$ using the density function for ρ. Note that the only difference between this system and the system in Section 4.1 is the constant multiplicative factor $2\sigma^2$ in the autocorrelation function. As it should, the FH system used over a Rayleigh fading channel exhibits time-selectivity in the same that the FH system used over an ideal channel does. If $T \le T_h$, the output spectrum has about the same bandwidth as the input signal spectrum. If $T > T_h$, the output signal spectrum has about the same bandwidth as the FH system power spectral density.

5.2 Rician Fading Channel

In addition to the receiver seeing a nonselective Rayleigh fading signal, there may be a nonfaded or "specular" component present. If the difference in propagation times of the nonfaded and faded signals is small compared with the minimum of the data bit duration and the dwell time, the overall channel is nonselective. Let the sum of the two components have amplitude S and phase Φ. The joint density of S and Φ is

$$f_{S,\Phi}(s,\phi) = \frac{s}{2\pi\sigma^2} \exp\left[- \frac{s^2 + A^2 - 2sA\cos\phi}{2\sigma^2} \right] ; \; s \geq 0, \; 0 \leq \phi \leq 2\pi \qquad (5.3)$$

where A is the amplitude of the nonfaded component and $2\sigma^2$ is the expected value of the amplitude squared of the faded component [23]. The fading channel is called Rician because the marginal distribution for S is given by the Rician density

$$f_S(s) = s/\sigma^2 \exp\left[-\frac{1}{2} (s^2 + A^2)/\sigma^2 \right] \; I_0(As/\sigma^2); \; s \geq 0 \qquad (5.4)$$

where I_0 is the zeroth order modified Bessel function. The output of the hopper and input to the channel is

$$x(\xi) = \text{Re} \left\{ \frac{1}{2} w(\xi) \; e^{-j2\pi f_h(\xi)\xi - j\alpha(\xi)} \right\} .$$

The channel output is then

$$y(\xi) = \frac{1}{2} \text{Re}\{(\rho e^{j\gamma} + A) w(\xi) \; e^{-j2\pi f_h(\xi)\xi - j\alpha(\xi)}\}$$

where

$$e^{j\gamma} + A = S \; e^{j\Phi} .$$

Assuming that $f_h(t) = f_d(t)$, the output of the dehopper is

$$s(t) = \mathrm{Re}\{\int g_{CBA}(t,\xi) w(t-\xi)\, d\xi\}$$

where

$$g_{CBA}(t,\xi) = \frac{1}{2} S e^{j\Phi} e^{-j\alpha(t)+j\beta(t)} \delta(\xi) \quad . \qquad (5.5)$$

The autocorrelation function is the same as in Section 4.1 except for the constant multiplicative factor $E\{S^2\} = 2\sigma^2 + A^2$.

5.3 Two Path Channel

Now, suppose we have a two path channel defined in the following manner. If the input to the channel is

$$x(t) = \mathrm{Re}\{r(t)\, e^{j2\pi f_0 t}\} \quad ,$$

then the output is

$$y(t) = \mathrm{Re}\{Ar(t-\tau_0)\, e^{j2\pi f_0 t} + Br(t)\, e^{j2\pi f_0 t}\} \qquad (5.6)$$

where τ_0 is the delay of the first component with respect to the second and A and B are complex numbers describing the amplitude and phase of the two received signal components. There is also a nominal delay time of the total signal which we ignore since it is only the relative delay that enters into the analysis. If τ_0 is sufficiently small compared with the minimum of T and T_h, the channel is nonselective. (A phase is introduced in the first component relative to the second component.) We examine what occurs when the condition on τ_0 is not necessarily met.

If the input to the channel is the signal from the hopper, then the output of the channel is

$$y(t) = \mathrm{Re}\left[\frac{A}{2}\, e^{-2\pi f_h(t-\tau_0)(t-\tau_0)-j\alpha(t-\tau_0)}\, e^{j2\pi f_c t}\, v(t-\tau_0)\right.$$
$$\left. + \frac{B}{2}\, e^{-j2\pi f_h(t)t-j\alpha(t)}\, e^{j2\pi f_c t}\, v(t)\right] .$$

At the dehopper, the signal is demodulated and sent through the bandpass filter with impulse response $h_W(t)$. As before, the filter bandwidth is smaller than the smallest separation between any two hopping frequencies. If we again assume that $f_d(t) = f_h(t)$, we find that the output of the FH system operating over the two path channel is

$$s(t) = \mathrm{Re}\left\{\int g_{CBA}(t,\xi)w(t-\xi)\, d\xi\right\}$$

where

$$g_{CBA}(t,\xi) = \frac{A}{4}\, e^{-j2\pi f_h(t-\xi)(t-\xi)+j2\pi f_d(t-\xi+\tau_0)(t-\xi+\tau_0)}\, e^{-j\alpha(t-\xi)+j\beta(t-\xi+\tau_0)}$$
$$\cdot\, e^{j2\pi f_c\tau_0}\, h_W(\xi-\tau_0) + \frac{B}{4}\,\delta(\xi)\, e^{-j\alpha(t-\xi)+j\beta(t-\xi)} \quad . \qquad (5.7)$$

To analyze this response, we need to consider the relationship between τ_0 and the FH system dwell time. If τ_0 is greater than T_h, the first term of (5.7) is equal to zero since $f_h(t) \neq f_d(t + \tau_0)$ for any time t. That is, the second term of (5.7) is the signal assumed to be synchronized with the dehopper, while the delayed first term is delayed enough so that it has no effect on the output. Thus, we have that

$$g_{CBA}(t,\xi) = \frac{B}{4}\,\delta(\xi)\, e^{-j\alpha(t)+j\beta(t)}$$

and

$$R_{g_{CBA}}(t,s;\ \xi,\zeta) = \begin{cases} \frac{B^2}{16}\,\delta(\xi)\delta(\xi-\zeta)\left[1 - \frac{|s-t|}{T_h}\right] & ;\ |s-t| \le T_h \\ 0 & ;\ \text{otherwise} \end{cases} .$$

If we assume that τ_0 is equal to some γ that is nearly zero relative to T_h, then $f_h(t) = f_d(t + \tau_0)$ for nearly all t. The result is a phase introduced in the first term of (5.7) with respect to the second term. The input delay-spread function is

$$g_{CBA}(t,\xi) = \frac{1}{4}\,\delta(\xi)\; e^{-j\alpha(t)+j\beta(t)}\left[Ae^{j\phi(t)} + B\right]$$

where

$$\phi(t) \simeq (2\pi f_c + 2\pi f_d(t))\tau_0 .$$

We assume that $\phi(t)$ is a random process analogous to $\alpha(t)$ and $\beta(t)$. That is, we assume that $\phi(t)$ is a uniformly distributed (on $[0,2\pi)$) random variable during each hopping interval independent of $\alpha(t)$ and $\beta(t)$. Then the autocorrelation function is

$$R_{g_{CBA}}(t,s;\ \xi,\zeta) = \begin{cases} \left[\left(\frac{B}{4}\right)^2 + \left(\frac{A}{4}\right)^2\right] \delta(\xi)\delta(\xi-\zeta)\left[1 - \frac{|s-t|}{T_h}\right] & ;\ |s-t| \le T_h \\ 0 & ;\ \text{otherwise} \end{cases}$$

where crossterms have disappeared due to the assumption on $\phi(t)$. If $A = B$, we see that the amplitude of the autocorrelation function when $\tau_0 \simeq 0$ is twice that of $R_{g_{CBA}}$ when $\tau_0 \ge T_h$. Both terms of the signal received at the dehopper are specular terms that are synchronized with the receiver. Note that if A and B are random variables, independent of the random starting time and the phases, then we can replace A^2 and B^2 with their expected values in the expressions for the autocorrelation functions.

If $\gamma < |\tau_0| < T_h$ for some $\gamma > 0$ that is small relative to T_h, then for a portion of the hopping interval there is a contribution from the first term since $f_h(t) = f_h(t + \tau_0)$ during part of each hopping interval.

In the remaining portion of the hopping interval, the first term is zero. Thus, for τ_0 in this range, the FH system introduces time-selectivity in the sense that an FSK signal entering the system sees one type of fading over one portion of the hopping interval and another type of fading over the remainder. For example, if the bit duration T is equal to the dwell time T_h, the time-selectivity introduced may cause the spectrum of the incoming signal to be spread up to two times its original width. If the system is a SFH system, the signals over the two paths interfere, possibly both constructively and destructively, during the portion of the hopping interval when both signals of the two path channel make a contribution. That is, there is an overlap of bits from the first part of each hopping interval with bits from the last part. The duration of each overlap depends on the magnitude of τ_0, and the position of the overlap within the hopping interval depends on the sign of τ_0. We see, for the two path channel, that the worst situation is when the delay of one path relative to the other path has magnitude greater than zero, but less than the dwell time.

Instead of one component of the two path channel being a distorted version of the output of the frequency-dehopper by a multiplicative factor B, we may consider it to be a superposition of many signals traveling different paths so that the resultant modulation is a time-varying random process. We do this in the next section.

6. WSSUS SELECTIVE FADING CHANNELS

In this section we consider the channel to be a WSSUS selective fading channel. The general model is for an input

$$x(t) = \mathrm{Re}\{r(t)e^{j2\pi f_0 t}\} \quad ,$$

the output is

$$y(t) = \mathrm{Re}\{A\, r(t-\tau_0)e^{j2\pi f_0 t} + \int g_B(t,\xi)r(t-\xi)e^{j2\pi f_0 t}d\xi\} \tag{6.1}$$

where $g_B(t,\xi)$ is a stationary complex zero mean Gaussian random process and τ_0 is the delay of the first component with respect to the second. We consider two possible values for A. If $A = 1$, then there is a specular component present in the output and the channel is a Rician fading channel. If $A = 0$, then there is no specular component and the channel is a Rayleigh fading channel. Since we are considering a WSSUS fading channel, the autocorrelation function for $g_B(t,\xi)$ has the form

$$R_{g_B}(t,t+\kappa;\xi,\zeta) = P_{g_B}(\kappa,\xi)\delta(\xi-\zeta) \ . \tag{6.2}$$

6.1 Rayleigh Fading Channel

We first consider the case $A = 0$. We have from (2.3) that

$$H_B(\tau,\eta) = g_B(\tau,\tau-\eta).$$

The composite time-varying impulse response for the FH system operating over a WSSUS Rayleigh fading channel is

$$H_{CBA}(t,\xi) = \iint H_C(t,\tau)H_B(\tau,\eta)H_A(\eta,\xi)d\eta d\tau$$

$$= \frac{1}{4} e^{-j2\pi f_h(\xi)\xi - j\alpha(\xi)} \int e^{j2\pi f_d(\tau)\tau + j\beta(\tau)} H_B(\tau,\xi)h_W(t-\tau)d\tau \ .$$

and from (2.3)

$$g_{CBA}(t,\xi) = \frac{1}{4} e^{-j2\pi f_h(t-\xi)(t-\xi) - j\alpha(t-\xi)} \int e^{j2\pi f_d(\tau)\tau + j\beta(\tau)} g_B(\tau,\tau-(t-\xi))h_W(t-\tau)d\tau. \tag{6.3}$$

To obtain the autocorrelation function of g_{CBA} defined as

$$R_{g_{CBA}}(t,s;\xi,\zeta) = E\{g_{CBA}{}^*(t,\xi)g_{CBA}(s,\zeta)\} \quad ,$$

we make the following assumptions:

(i) $\alpha(t)$ is uniformly distributed on $[0,2\pi)$ each hopping interval and is independent of $\alpha(t)$ any other hopping interval.

(ii) $\beta(t)$ satisfies (i).

(iii) There is a random starting time t_0 for the hopping pattern which is uniformly distributed on $[0,T_h)$.

(iv) $\alpha(t)$, $\beta(t)$ and $g_B(t,\xi)$, are mutually independent random processes and are independent of the random variable t_0.

Also, given (6.2) we have

$$E\{g_B^*(\tau,\tau-(t-\xi))g_B(\nu,\nu-(s-\zeta))\} = P_{g_B}(\nu-\tau;\tau-(t-\xi))\delta(\nu-(s-\zeta)-(\tau-(t-\xi))).$$

So the autocorrelation function for the whole system becomes

$$R_{g_{CBA}}(t,s;\xi,\zeta) = \frac{1}{16}\iint P_{g_B}(\nu-\tau;\tau-(t-\xi)\delta(\nu-(s-\zeta)-(\tau-(t-\xi)))E\{\Omega\}h_W(t-\tau)h_W(s-\nu)d\tau d\nu \tag{6.4}$$

where the expectation is

$$E\{\Omega\} = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} E\{e^{j2\pi f_n(t-\xi-(s-\zeta))} e^{-j2\pi f_m(\tau-\nu)} p_{T_h}(t-\xi-t_0-nT_h)$$

$$\cdot\, p_{T_h}(s-\zeta-t_0-nT_h)p_{T_h}(\tau-t_0-mT_h)p_{T_h}(\nu-t_0-mT_h)\} \, .$$

If we assume that the hopper and dehopper frequency sequences are synchronized so that $m = n$ and that there is no frequency offset between them, then $f_m = f_n$. Using these assumptions and making use of the δ-function in the integrand of (6.4), the expectation simplifies to

$$E\{\Omega\} = \frac{1}{T_h}\int p_{T_h}(t-\xi-v)p_{T_h}(s-\zeta-v)p_{T_h}(\tau-v)\; p_{T_h}(\nu-v)\; dv.$$

Let $u = t-\xi-v$ and note that the product of the four rectangular pulses can be simplified since

$$p_{T_h}(u)p_{T_h}(\nu-\tau+u) = p_{T_h-|\nu-\tau|}(u+\min(0,\nu-\tau)); \;\; |\nu-\tau| \le T_h$$

and

$$p_{T_h}(\tau-(t-\xi)+u)p_{T_h}(\nu-(t-\xi)+u) = p_{T_h-|\nu-\tau|}(u+\min(\nu,\tau)-(t-\xi)); \;\; |\nu-\tau| \le T_h.$$

If $\nu-\tau > 0$, the integral becomes

$$E\{\Omega\} = \frac{1}{T_h}\int p_{T_h-|\nu-\tau|}(u)p_{T_h-|\nu-\tau|}(u+\tau-(t-\xi))du$$

$$= 1 - \frac{|\nu-\tau|+|\tau-(t-\xi)|}{T_h} \;\; ; \;\; |\tau-(t-\xi)| \le T_h-|\nu-\tau|$$

which is also the result when $\nu-\tau \le 0$. Finally the system autocorrelation function in (6.4) becomes

$$R_{g_{CBA}}(t,s;\xi,\zeta) = \begin{cases} \frac{1}{16}\iint P_{g_B}(\nu-\tau;\tau-(t-\xi))\delta(\nu-(s-\zeta)-\tau+(t-\xi))\left[1-\frac{|\nu-\tau|+|\tau-(t-\xi)|}{T_h}\right] \\ \quad \cdot h_W(t-\tau)h_W(s-\nu)d\tau d\nu; \ |\tau-(t-\xi)|+|\nu-\tau| \le T_h \\ 0 \qquad ; \text{ otherwise} . \end{cases} \tag{6.5}$$

This correlation function is doubly-selective since the fading channel is doubly-selective. It is not clear from (6.5) whether or not the composite process g_{CBA} is WSSUS. We analyze the cases when the fading channel is purely time-selective or purely frequency-selective next to see if the resulting system is WSSUS.

6.1.1 Purely Time-Selective Fading Channel

Assume that the WSSUS channel is purely time-selective. Then,

$$P_{g_B}(\nu-\tau;\tau-(t-\xi)) = P_{g_B}(\nu-\tau;0)\delta(\tau-(t-\xi)) = P_B(\nu-\tau)\delta(\tau-(t-\xi))$$

so that the system autocorrelation function in (6.5) simplifies to

$$R_{g_{CBA}}(t,s;\xi,\zeta) = \frac{1}{16}\iint P_B(\nu-\tau)\delta(\tau-(t-\xi))\delta(\nu-(s-\zeta))\left[1-\frac{|\nu-\tau|}{T_h}\right]h_W(t-\tau)h_W(s-\nu)d\tau d\nu; \quad |\nu-\tau| \le T_h$$

$$= \begin{cases} \frac{1}{16} P_B(s-t-(\zeta-\xi))\left[1-\frac{|s-t-(\zeta-\xi)|}{T_h}\right]h_W(\xi)h_W(\zeta); \ |s-t-(\zeta-\xi)| \le T_h \\ 0 \qquad ; \text{ otherwise} . \end{cases} \tag{6.6}$$

This result implies that the FH system using a WSSUS purely time-selective channel is WSS since the autocorrelation function depends on the time difference s-t only. That is, the series connection of the three individually WSS systems, frequency-hopper, WSSUS time-selective channel, and

frequency-dehopper, retains the WSS property. However, the system does not have the mathematical form for the US channel. Thus, the complex modulation $g_{CBA}(t,\cdot)$ corresponding to different path delays are correlated in some manner. We see that if the filter $h_W(\cdot)$ has the form of an impulse in time that the US property would be present. The loss of this property is not of great importance because uncorrelated scattering has no physical interpretation for the FH system itself. Note also that the system is no longer purely time-selective. These results are a consequence of the finite bandwidth of the bandpass filter.

Suppose the bandpass filter, which is assumed to have a transfer function that is nearly flat over its entire bandwidth, has infinite bandwidth. For example, if $h_W(\xi)$ is ideal as in (3.3) we see that

$$\begin{aligned}\lim_{W\to\infty} h_W(\xi)h_W(\zeta) &= \lim_{W\to\infty} W\mathrm{sinc}(W\xi)W\mathrm{sinc}(W\zeta)\cos(2\pi f_c\xi)\cos(2\pi f_c\zeta)\\ &= \delta(\xi)\delta(\zeta)\\ &= \delta(\xi)\delta(\xi-\zeta)\end{aligned}$$

where we have used properties of the δ-function in (4.3) and (4.4) assuming the δ-function product appears only as an integrand of a double integral. Thus,

$$\lim_{W\to\infty} R_{g_{CBA}}(t,s;\xi,\zeta) = \begin{cases} \frac{1}{16} P_B(s-t)\left[1 - \frac{|s-t|}{T_h}\right]\delta(\xi)\delta(\xi-\zeta); & |s-t| \le T_h \\ 0 & ;\ \text{otherwise.} \end{cases} \tag{6.7}$$

This is a correlation function for a WSSUS channel that is purely time-selective. The loss of the purely time-selective property when the

bandpass filter bandwidth is finite is not surprising. Notice that the bandpass filter adds frequency-selectivity to the channel since frequencies of the signal that fall within the passband are "faded" differently than the frequencies that lie outside the passband. Hence, frequency-selectivity is evident trivially when we have a finite bandwidth bandpass filter and we look at all frequencies. Locally, within the passband of the filter, the channel can be considered purely time-selective. We use the autocorrelation function for the wideband system, keeping in mind that the signal frequencies at the output of the frequency dehopper lie within the band of width W.

Examining (6.7), we see that there is additional Doppler spread of the input signal due to the fading channel itself. If $T \leq T_h$, the Doppler spread is approximately the bandwidth of the Fourier transform of $P_B(\kappa)$. If the bit duration is greater than the dwell time, the output spectrum has approximately the same bandwidth as the power spectral density

$$\mathcal{P}_{CBA}(f) = \int_{-T_h}^{T_h} P_B(\kappa) \left[1 - \frac{|\kappa|}{T_h}\right] e^{-j2\pi f\kappa}\, d\kappa \;.$$

If the correlation duration of the channel is much greater than the correlation duration of the communication system (the dwell time), then the system causes most of the spread. Alternately, if the correlation duration of the channel is smaller than that of the system, the channel is responsible for most of the spread. In either case, when $T < T_h$, the Doppler spread is less than the bandwidth of the Fourier transform of $P_B(\kappa)$ plus $(c/T)[T/T_h - 1]$. Spreading of the spectrum degrades the total system performance independent of the relationship between T and T_h since energy is spread wider than the passband W so that less signal energy is detected at the FSK receiver.

6.1.2 Purely Frequency-Selective Fading Channel

Assume that the WSSUS channel is purely frequency-selective so that

$$P_{g_B}(\nu-\tau;\tau-(t-\xi)) = Q_B(\tau-(t-\xi)).$$

Then the autocorrelation function in (6.5) becomes

$$R_{g_{CBA}}(t,s;\xi,\zeta) = \begin{cases} \frac{1}{16}\int Q_B(x)\left[1-\frac{|s-t-(\zeta-\xi)|+|x|}{T_h}\right]h_W(\xi-x)h_W(\zeta-x)\,dx\ ; & |x|\le T_h-|s-t-(\zeta-\xi)| \\ 0 & ;\ \text{otherwise} \end{cases} \tag{6.8}$$

where $x = \tau - (t-\xi)$. This autocorrelation function depends on the time difference s-t only so that the FH system using a WSSUS purely frequency-selective channel is WSS. We expect the system to be doubly-selective since the channel is frequency-selective and the FH system is time-selective. Again we let the bandwidth of the bandpass filter go to ∞ so that the frequency-selectivity caused by the filter goes away. Then

$$\lim_{W\to\infty} R_{g_{CBA}}(t,s;\xi,\zeta) = \frac{1}{16}\int Q_B(x)\left[1-\frac{|s-t-(\zeta-\xi)|+|x|}{T_h}\right]\delta(\xi-x)\delta(\zeta-x)\ ;\quad |x|\le T_h-|s-t-(\zeta-\xi)|$$

$$= \begin{cases} \frac{1}{16}Q_B(\xi)\left[1-\frac{|s-t|+|\xi|}{T_h}\right]\delta(\xi-\zeta); & |\xi|+|s-t|\le T_h \\ 0 & ;\ \text{otherwise} \end{cases} \tag{6.9}$$

where we use the property

$$\int f(x)\,\delta(\alpha-x)\delta(x-\beta)\,dx = f(\alpha)\delta(\alpha-\beta) \quad . \tag{6.10}$$

The limit in (6.9) is a correlation function for a doubly-selective WSSUS channel where the frequency-selectivity is introduced by the fading channel rather than by the bandpass filter. We calculate the multipath spread using

$$Q_{CBA}(\xi) = \frac{1}{16}\, Q_B(\xi) \left[1 - \frac{|\xi|}{T_h}\right] \; ; \; |\xi| \le T_h \tag{6.11}$$

obtained by letting $s = t$ in $P_{g_{CBA}}(s-t;\xi)$. Consider the delay density spectrum $Q_{CBA}(\xi)$ when $Q_B(\xi)$ is triangular with base $2T_B$. Fix the dwell time and first consider when $T_B > T_h$. From (6.11) we see that the multipath spread is less than $2T_h$ as long as no two consecutive hopping frequencies are equal. If $T \ge T_h$ there is no intersymbol interference, but since each bit received is spread in time, less energy per bit is detected at the receiver. When $T < T_h$ intersymbol interference is produced among the T_h/T bits within a hopping interval, while bits from other hopping intervals do not interfere. Thus, when the correlation duration of the channel is larger than the dwell time, we see that the FH system limits the amount of multipath spread to be less than $2T_h$ and it limits the occurrence of intersymbol interference.

Next we consider when $T_B < T_h$ with the dwell time fixed as before. The multipath spread of the system is approximately the multipath spread of the fading channel. Since this is less than $2T_h$, the performance of the system is improved compared to that of the previous case. That is, the performance of the system is better when the frequency-selectivity of fading channel is limited. Note that the only time-selectivity introduced

is from the FH system. Also, note the tradeoff between using a FFH system versus a SFH system. E.g., a FFH system helps combat frequency-selective fading, but it introduces time-selective fading. A SFH system does not introduce time-selective fading, but it is not as effective against preventing intersymbol interference caused by a frequency-selective channel.

6.2 Rician Fading Channel

In this section we consider when $A = 1$ in (6.1). The specular term arrives at the receiver with a delay τ_0 relative to the faded term. If this delay is much less than the minimum of T and T_h, then the first term of $y(t)$ in (6.1) may be approximated by $r(t)\ e^{j2\pi f_0 t + j\phi}$. For the FH system operating over this channel, the input delay-spread function is

$$g_{CBA}(t,\xi) = \frac{1}{4}\, e^{-j2\pi f_h(t-\xi)(t-\xi) - j\alpha(t-\xi)} \{e^{j2\pi f_d(t-\xi)(t-\xi) + j\beta(t-\xi)}\ e^{j\phi}\ h_W(\xi)$$

$$+ \int e^{j2\pi f_d(\tau)\tau + j\beta(\tau)}\ g_B(\tau, \tau-(t-\xi))\ h_W(t-\tau)\ d\tau\} \quad .$$

In general, the autocorrelation consists of four terms, but we assume $g_B(t,\xi)$ to be zero mean so that the crossterms are zero. Hence, we have that the autocorrelation function $R_{g_{CBA}}(t,s;\xi,\zeta)$ for the FH system using a WSSUS channel is the sum of the autocorrelation functions found in (4.1) and (6.1). As in Section 5.2, when the magnitude of τ_0 is greater than the dwell time, the specular term is zero. When $|\tau_0| < T_h$, there is a contribution from the first term since for a portion of each hopping interval, $f_h(t-\tau_0) = f_d(t)$. Thus, a Rician fading channel introduces additional time-selectivity because of the presence of two versions of the transmitted signal at the receiver.

7. CORRELATION BETWEEN FREQUENCY SLOTS

In this section we investigate the correlation between signals transmitted in adjacent frequency slots with separation Δ' and determine the separation necessary to guarantee that the fading of these signals have correlation less than some positive number β. We assume that the fading channel is purely frequency-selective.

Suppose that we have two transmitters each consisting of an FSK modulator and a bandpass filter. We denote the carrier frequencies f_1 and f_2, and assume that the filter passes only the difference frequency. The transmitted signals pass through a fading channel with time-varying impulse response $g_B(t,\xi)$. The crosscorrelation function of the input delay-spread function for the wideband system using (4.3) is

$$R_{g_{BA_1} g_{BA_2}}(t,s;\xi,\zeta) = E\{g_{BA_1}^*(t,\xi)\, g_{BA_2}(s,\zeta)\}$$

$$= \frac{1}{4} Q_B(\xi)\, e^{-j2\pi f_2(s-\zeta)+j2\pi f_1(t-\xi)} \delta(\xi-\zeta) \qquad (7.1)$$

where

$$g_{BA_i}(t,\xi) = \frac{1}{2} e^{-j2\pi f_i(t-\xi)}\, g_B(t,\xi) \qquad \text{for } i = 1,2.$$

Define the frequency correlation function (See (2.47) and (2.75) in [6])

$$R_{f_1 f_2}(\ell,f) = \iiiint R_{g_{BA_1} g_{BA_2}}(t,s;\xi,\zeta)\, e^{-j2\pi(fs-\ell t)}\, d\xi\, d\zeta\, dt\, ds$$

$$= \iiint Q_B(\zeta)\, u_1^*(t-\xi)\, u_2(s-\xi)\, e^{-j2\pi(fs-\ell t)}\, d\xi\, dt\, ds \qquad (7.2)$$

where $u_i(x) = 1/2\, e^{-j2\pi f_i x}$ when (7.1) is substituted into (7.2). Thus, the frequency correlation function of the outputs of a purely frequency-selective WSSUS channel when the input envelopes are pure frequency tones is

$$R_{f_1f_2}(\ell,f) = \delta(f+f_2)\delta(\ell+f_1)\,\frac{1}{4}\int Q_B(\xi)\; e^{-j2\pi(f_1-f_2)\xi}\, d\xi \quad . \tag{7.3}$$

Suppose $Q_B(\xi)$ is of the form $\sin(\pi\lambda\xi)/(\pi\xi)$ with $\lambda = 2/T_M$ so that the Fourier transform is

$$\int Q_B(\xi)\; e^{-j2\pi(f_1-f_2)\xi}\, d\xi = \begin{cases} 1 \;\; ; & |f_1 - f_2| \le \dfrac{1}{T_M} \\ 0 \;\; ; & \text{otherwise.} \end{cases}$$

Then (7.3) becomes

$$R_{f_1f_2}(\ell,f) = \frac{1}{4}\,\delta(f+f_2)\delta(\ell+f_1) \qquad ; \; |f-\ell| \le \frac{1}{T_M}$$

$$= \begin{cases} \dfrac{1}{4}\,\delta(f+f_2)\delta(\ell+f_1) \; ; & \Delta' \le \dfrac{1}{T_M} \\ 0 \; ; & \text{otherwise.} \end{cases} \tag{7.4}$$

We note that if $\Delta' < 1/T_M$, then the two transmitted frequency tones fade in a correlated manner. If the spacing between the tones is greater than the coherence bandwidth $(1/T_M)$, then the fading of the tones is uncorrelated.

Next consider the fading of signals in adjacent hopping intervals. Let

$$u_1(t) = \frac{1}{2}\, e^{-j2\pi f_1 t}\; p_{T_h}(t)$$

$$u_2(t) = \frac{1}{2}\, e^{-j2\pi f_2 t}\; p_{T_h}(t-T_h) \quad . \tag{7.5}$$

Substituting (7.5) into (7.2) gives

$$R_{f_1 f_2}(\ell,f) = \frac{T_h^2}{4}\,\mathrm{sinc}T_h(f+f_2)\,\mathrm{sinc}T_h(\ell+f_1)\exp[j\pi T_h(3(\ell+f_1)-(f+f_2)]$$

$$\cdot \int Q_B(\xi)\,\exp[-j2\pi(f-\ell)\xi]\,d\xi \qquad (7.6)$$

for the frequency correlation function between the two adjacent signals. We consider the magnitude of this in the following analysis so the complex exponential containing the information of when the signals are sent does not contribute anything. Thus, the following results give the correlation in fading between signals sent in any two hopping intervals. For example, the signals may be sent in two different hopping intervals by the same transmitter or they may be sent from two different transmitters that form part of a multiple-access system. The crosscorrelation function of the faded signals depends on the dwell time, the two carrier frequencies f_1 and f_2, and the delay density spectrum Q_B of the fading channel. If we assume that $Q_B(\xi)$ is a sinc function with first zero crossing at $T_M/2$, then the magnitude of (7.6) becomes

$$\left|R_{f_1 f_2}(\ell,f)\right| = \begin{cases} \dfrac{T_h^2}{4}\left|\mathrm{sinc}T_h(f+f_2)\;\mathrm{sinc}T_h(\ell+f_1)\right| & ;\ |f-\ell| \le \dfrac{1}{T_M} \\ 0 & ;\ \text{otherwise.} \end{cases} \qquad (7.7)$$

Note that as $T_h \to \infty$, (7.7) converges to (7.4). The correlation function in (7.7) is zero if either of the quantities $f+f_2$ or $\ell+f_1$ is an integer. It is also zero when the two frequencies f and ℓ are further apart than $1/T_M$. The frequencies f and ℓ are the arguments of the crosscorrelation function; we need to find what Δ' should be for the crosscorrelation function to be zero for any f and ℓ. If $\Delta' = |f_1-f_2| >> 2/T_h + 1/T_M$, then

the frequency components of the signal centered at f_1 fade independently of the frequency components of the signal centered at f_2. This condition may be difficult to achieve so we consider smaller differences between the two frequencies. We find how large Δ' should be for the magnitude of the correlation (normalized by the factor $T_h^2/4$) between signals to be less than some positive number β. To do this, suppose that $f = \ell$ and consider when the condition

$$|\mathrm{sinc}[T_h(f + f_1 + \Delta')]\ \mathrm{sinc}[T_h(f + f_1)]| \leq \beta$$

is met. (Note that here we assume $f_2 > f_1$ so that $f_2 = f_1 + \Delta'$.) We should have

$$\Delta' > \frac{1}{T_M} + \frac{x}{T_h}$$

where we find $x > 0$ from

$$|\mathrm{sinc}[T_h(f + f_1) + x]\ \mathrm{sinc}[T_h(f + f_1)]| \simeq \beta$$

such that for all $y > x$

$$|\mathrm{sinc}[T_h(f + f_1) + y]\ \mathrm{sinc}[T_h(f + f_1)]| \leq \beta \ .$$

In Table 1, we list x for given values of $T_h(f - f_1)$ and β. We note that as $T_h(f + f_1)$ increases, the necessary spacing between frequency slots for signals in the slots to fade in an uncorrelated manner decreases. Of course, as β, the largest (normalized) correlation allowed, decreases the frequency separation necessary increases. See Figure 2 for a graph of β versus x for $f + f_1$ equal to zero. The steps in the curve, corresponding to jumps in the frequency separation necessary for normalized

Table 1. Values of x for given values of $T_h(f + f_1)$ and β.

$T_h(f + f_1)$	$\beta = .5$	$\beta = .2$	$\beta = .05$
0.0	.6034	1.5620	5.6523
0.1	.5973	1.5480	5.6428
0.2	.5780	.8155	5.6092
0.3	.5420	.8007	4.6735
0.4	.4810	.7769	4.5969
0.5	.3739	.7396	3.6442
0.6	.0745	.6792	2.6850
0.7	.0000	.5710	1.7346

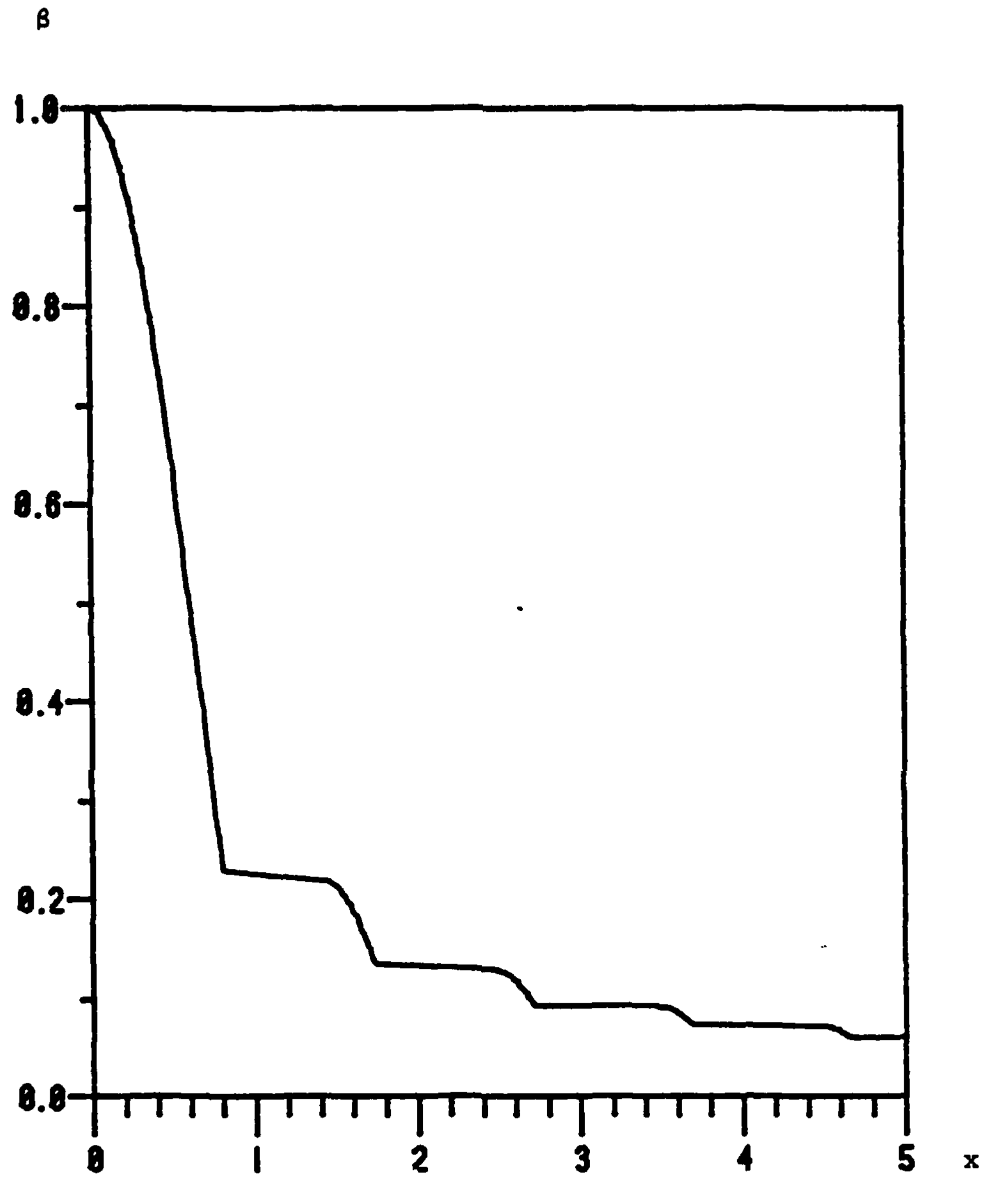


Figure 2. β versus x given that $(f + f_1) = 0$.

correlation in fading to be less than β, are due to the lobes in the magnitude of the correlation function. Curves for larger $T_h(f + f_1)$ have smaller amplitudes than the curve in Figure 2, and are shifted to the left by $-(f + f_1)$. If we have that $f \neq \ell$, the separation Δ' may be smaller by a fraction of $1/T_M$ than the separation necessary when $f = \ell$. In other words, if $\Delta' > 1/T_M + x/T_h$, then the normalized correlation in fading between any frequency component of one signal and any frequency component of the other signal is less than β.

8. CONCLUSIONS

The system consisting of a frequency-hopper, WSSUS fading channel, and a frequency-dehopper in series was modeled as a fading channel seen by an FSK system. It was found that a noncoherent FH system can be characterized as a WSSUS time-selective channel. The degree of time-selectivity introduced by the system depends on the relationship between the dwell time and the duration of a data bit. An FFH system introduces more spreading of the spectrum of a signal than a SFH system.

A constant detuning factor between the frequencies of the hopper and dehopper was found to introduce a Doppler shift equal in magnitude to this factor. If the detuning factor is a function of frequency, the FH system is no longer wise-sense-stationary (for a deterministic hopping pattern). However, if the hopping pattern is random with a uniform probability distribution, the system is wide-sense stationary. Given that the detuning factor is a fraction λ of the instantaneous frequency of the signal, the mean Doppler spread is λ times the total system bandwidth and the mean Doppler shift is $-\lambda$ times the average frequency of the signal.

The autocorrelation functions were calculated for the FH system operating over nonselective fading channels. For the nonselective Rayleigh and Rician fading channels, it was found that the system autocorrelation functions are the same as the autocorrelation function for the FH system operating over an ideal channel except for constant multiplicative factors. FH communication over a two path channel was investigated.

The autocorrelation function was calculated for the FH system in series with a WSSUS channel, and the system was found to no longer be

wide-sense-stationary. Time-selective and frequency-selective fading channels were considered separately. To obtain wide-sense stationary systems, we let the bandwidth of the output bandpass filter of the frequency-dehopper go to infinity. Both Rayleigh and Rician selective fading channels were studied. It was found that a Rician selective fading channel introduces time-selectivity (in addition to that introduced by a Rayleigh fading channel and the FH system) if the relative delay between the two signal components is greater than zero, but less than the dwell time. An FFH system can tolerate frequency-selective fading better than a SFH system. The effects of intersymbol interference introduced to a communication system by a frequency-selective channel are reduced when a SFH system is used. Intersymbol interference is eliminated when an FFH system is used.

A frequency crosscorrelation function was calculated for the two output signals from a purely frequency-selective fading channel with two input signals at different carrier frequencies. This correlation function was used to find the magnitude of the correlation between fading signals in different frequency slots. The frequency spacing between adjacent frequency slots necessary for the fading of signals occupying these slots to have correlation less than some positive number was found. This was done for a particular channel delay density spectrum. The necessary separation depends on the hopping rate and the coherence bandwidth of the frequency-selective channel.

REFERENCES

1. M. Abramoivitz and I. A. Stegun, Handbook of Mathematical Functions, pp. 238-243, Dover Publications, Inc., New York, 1972.

2. F. Amoroso, "The Bandwidth of Digital Data Signals," IEEE Communication Magazine, vol. 18, pp. 13-24, November, 1980.

3. P. A. Bello, "Characterization of Randomly Time-Variant Linear Channels," IEEE Transactions on Communication Systems, vol. CS-11, pp. 360-393, December, 1963.

4. P. A. Bello, "Some Techniques for the Instantaneous Real-Time Measurement of Multipath and Doppler Spread," IEEE Transactions on Communication Technology, vol. 13, pp. 285-292, September, 1965.

5. P. A. Bello and B. D. Nelin, "The Influence of Fading Spectrum on the Binary Error Probabilities of Incoherent and Differentially Coherent Matched Filter Receivers," IRE Transactions on Communication Systems, vol. 10, pp. 160-168, June, 1962.

6. D. E. Borth, "Introduction to Fading Channels," Chapter 2 of Coordinated Science Laboratory Report R-880 (UILU-ENG80-2212), University of Illinois, April, 1980.

7. F. D. Garber and M. B. Pursley, "Effects of Time-Selective Fading on Slow-Frequency-Hopped DPSK Spread-Spectrum Communications," 1981 National Telecommunications Conference, Conference Record, vol. 4, pp. G8.1.1-G8.1.5, November, 1981.

8. F. D. Garber and M. B. Pursley, "Effects of Frequency-Selective Fading on Slow-Frequency-Hopped DPSK Spread-Spectrum Multiple-Access Communications," 1982 IEEE Military Communications Conference, Progress in Spread-Spectrum Communications, vol. 2, pp. 35.2-1-35.2-6, October, 1982.

9. E. A. Geraniotis and M. B. Pursley, "Error Probabilities for Slow-Frequency-Hopped Spread-Spectrum Multiple-Access Communications over Fading Channels," IEEE Transactions on Communications, vol. COM-30, pp. 996-1009, May, 1982.

10. E. A. Geraniotis, "Analysis of Binary Frequency-Hopped Spread-Spectrum Multiple-Access Communication Systems." M. S. Thesis, University of Illinois, July, 1980.

11. E. A. Geraniotis, "Coding for BPSK SFH/SSMA Communication over Fading Channels," 1982 IEEE Military Communications Conference, Progress in Spread-Spectrum Communications, vol. 2, pp. 35.1-1 - 35.1-5, October, 1982.

12. I. S. Gradshteyn and I. M. Ryzhik, Tables of Integrals, Series, and Products, Academic Press, New York, 1980.

13. T. Kailath, "Channel Characterization: Time Variant Dispersive Channels," Chapter 6 of Lectures on Communication System Theory, pp. 95-123, McGraw-Hill, New York, 1961.

14. R. S. Kennedy, Fading Dispersive Communication Channels, pp. 9-67, Wiley, New York, 1969.

15. R. W. Nettleton and G. R. Cooper, "Performance of a Frequency Hopped Differentially Modulated Spread Spectrum Receiver in a Rayleigh Fading Channel," IEEE Transactions on Vehicular Technology, vol. VT-30, pp. 14-29, February, 1981.

16. A. Papoulis, Probability, Random Variables, and Stochastic Processes, pp. 444-445, McGraw-Hill, New York, 1965.

17. A. Papoulis, The Fourier Integral and its Applications, pp. 269-282, McGraw-Hill, New York, 1962.

18. M. B. Pursley, "Models for Selective Fading," Unpublished Report, University of Illinois, July, 1980.

19. H. Sherman, "Channel Characterization: Rapid Multiplicative Perturbations," Chapter 5 of Lectures on Communication System Theory, pp. 71-94, McGraw-Hill, New York, 1961.

20. H. Stark and F. B. Tuteur, Modern Electrical Communications, pp. 58-60, 428, Prentice-Hall, Inc., Englewood Cliffs, 1979.

21. S. Stein, Part III of Communication Systems and Techniques, McGraw-Hill, New York, 1966.

22. J. B. Thomas, Statistical Communication Theory, pp. 582-288, Wiley, New York, 1969.

23. G. L. Turin, "Error Probabilities for Binary Symmetric Ideal Reception through Nonselective Slow Fading and Noise," Proceedings of the IRE, vol. 46, pp. 1603-1619, September, 1958.

24. L. Zadeh and C. A. Dosoer, Linear System Theory, pp. 393-396, McGraw-Hill, New York, 1963.

END

FILMED

3-83

DTIC